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Dual-Ascent Procedures for Multicommodity Location/Allocation Problems with Balancing Requirements

by

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ABSTRACT

The multicommodity location–allocation problem with balancing requirements is related to one of the major logistics issues faced by distribution and transportation firms: the management of the fleet of vehicles over a medium to long term planning horizon. We present a dual-ascent approach for this formulation which may be used either to obtain a good heuristic solution, or to compute sharp bounds in a branch-and-bound scheme. We present and compare two variants of this approach, and analyze their performances both on a large number of randomly generated problems and on a large scale application to the strategic/tactical planning of the land operations of an international maritime container shipping firm. Our results demonstrate that a dual-ascent approach is highly effective in this context.

KEY WORDS: DUAL-ASCENT ALGORITHMS,
MULTICOMMODITY LOCATION–ALLOCATION MODELS,
EMPTY FLOW BALANCING, FLEET MANAGEMENT,
CONTAINER TRANSPORTATION.

RESUME

Le problème de localisation–allocation avec échanges inter–dépôts (ou exigences d'équilibrage) est associé à un des aspects importants de la planification logistique des entreprises de transport et de distribution, à savoir la gestion de la flotte de véhicules à moyen et long termes. Nous présentons une approche algorithmique de type ''dual ascendant'' qui peut être utilisée soit per se pour obtenir une bonne solution approchée, soit pour calculer des bornes efficaces dans un algorithme d'énumération de type ''branch-and-bound''. Nous présentons et comparons deux variantes de cette approche, et nous analysons leur comportement à partir d'un ensemble de problèmes générés aléatoirement et d'une application de grande taille au problème de planification stratégique/tactique des opérations terrestres d'une compagnie maritime internationale de transport de conteneurs. Les résultats obtenus montrent que l'approche duale ascendante est extrêmement efficace dans ce contexte.

MOTS CLES: ALGORITHMES DUAUX-ASCENDANTS,
MODELES DE LOCALISATION–ALLOCATION MULTIPRODUITS,
GESTION DE FLOTTE, MOUVEMENTS A VIDE,
TRANSPORT DES CONTENEURS.

INTRODUCTION

The management of the fleet of vehicles over a medium to long term planning horizon constitutes one of the major logistics issues faced by distribution and transportation firms. This strategic/tactical planning problem is particularly concerned with the location of the depots for the rolling stock and the forecast and management of the empty movements.

Maritime container transportation companies face a particularly challenging version of this problem when contemplating the planning of their land operations. Schematically, (for a full description and discussion, see Dejax, Crainic and Delorme, 1991) this problem may be stated as follows: Once a ship arrives at port, the shipping company has to deliver loaded containers to their designated in-land destinations. These containers come in several types and sizes, and various transportation modes may be used to move them. Following their delivery and unloading by the importing customer, empty containers may be returned to the depot of the port that initiated the loaded movement. However, they may also be repositioned to a different depot in prevision of future requests for empty containers. Indeed, the same, or other, customers are also requesting empty containers, of specific types, for subsequent shipping of their own products. Empty containers are delivered from depots, are loaded, and are subsequently moved, by land and maritime transportation means, to their final destinations.

A major complication of the problem is a consequence of the regional unbalances in empty container availabilities and needs throughout the network: some areas lack containers of certain types, while others have surpluses of them. This requires that empty containers be shipped between depots to "balance" the network. Note that interdepot transportation is assumed to be more efficient, per moved unit, than movements between depots and customers, due to the larger volumes of traffic which may be consolidated (e.g. unit trains) for transshipment over relatively long distances. This also justifies the very existence of interdepot movements (otherwise, empty containers would be positioned directly to the requesting depots), and thus differentiates this problem from a classical location/allocation application.

The general problem is therefore to locate the empty container depots in order to collect the supply of empty containers available at customers' sites and to satisfy the customer requests for empty containers, while minimizing the total operation cost: the cost of opening and operating the depots, plus the transportation costs between customers and depots and the

costs generated by moving empty containers among depots to "balance" the network. It may be interesting to note that, in practice, this strategic/tactical logistics problem may have to be repeatedly solved since container shipping companies do not often build their own depots; they rather use facilities available from other modes (e.g. ports and rail yards). It is then a question of deciding which facilities to use, according to the demand and the cost structure particular to the specific planning period. It is noteworthy that this last characteristic represents another difference with the usual location-allocation application, and that it also explains why, in practice, fixed depot costs may be rather low.

We have already proposed (Crainic, Dejax and Delorme, 1989) a modeling framework for this class of problems, that we call the *simple multicommodity location-allocation problem with balancing requirements (SLBP)*. Here, *simple* does not imply a "single echelon" formulation since the problem may be actually described as being "two echelon". It rather refers to the usual hypotheses concerning this problem, which assume that depot capacity does not restrict depot utilization, that transportation and depot operation costs are linear, and that supplies and demands are deterministic. (The first assumption may be easily verified in actual practice, while the second reflects the data and costing definitions which are currently available in industry.) The presence of balancing requirements represents one of the main characteristics of the problem, since it unveils a network flow structure that is most favorable to efficient algorithmic developments. It also constitutes a major difference between this formulation and classical location models, as reflected in the behaviour of the branch-and-bound algorithm (Crainic, Delorme and Dejax, 1991).

In this paper we present a dual-ascent based approach for solving simple multicommodity location-allocation problems with balancing requirements. In the first section, we recall the general formulation and some important properties, and briefly mention previous algorithmic approaches proposed for this problem. Section II is dedicated to the presentation of the dual-ascent approach. First the dual formulation of the strong linear relaxation of the SLBP is stated. Two subproblems are then derived, by fixing suitably chosen dual variables. Each of this subproblem allows us to compute lower and upper bounds on the optimal solution of the SLBP, as well as to derive a feasible solution for the multicommodity location-allocation problem with balancing requirements. Based on these results, two dual-ascent procedures are presented which may be used either to obtain a heuristic solution of the

SLBP, or to compute sharp bounds in a branch-and-bound scheme. We tested these procedures both on a wide gamut of randomly generated problems and on a large scale application. Section III presents the results of this experimentation.

In the past, dual-ascent methods have proved successful for several difficult problems; see Erlenkotter (1978), Van Roy and Erlenkotter (1982), Klincewicz and Luss (1987), Wong (1984), and Balakrishnan, Magnanti and Wong (1989), for example. Our results prove that this approach is equally effective for large scale, multicommodity location-allocation problems with balancing requirements.

I. THE MODEL

Let C, D and P be the sets of customers (cardinality m), candidate depots (cardinality n) and products (cardinality l), respectivelly. Let

f_j : "Fixed" cost of depot j, $j \in D$;

c_{ijp}, c_{jip} : Unit transportation costs (in both directions) for product p, $p \in P$, between customer i, $i \in C$, and depot j, $j \in D$;

s_{jkp}, s_{kjp} : Unit transportation costs for product p, $p \in P$, between depots j and k, $j \in D$, $k \in D$;

O_{ip} : Supply at node (customer) i, $i \in C$, of product p, $p \in P$;

D_{ip} : Demand at node (customer) i, $i \in C$, of product p, $p \in P$.

Define the following decision variables:

$y = (y_j)$: Opening vector for depots, $j \in D$; y_j equals 1 if depot j is open and zero otherwise;

x_{ijp}, x_{jip} : Flows (in both directions) of product p, $p \in P$, between customer i, $i \in C$, and depot j, $j \in D$;

w_{jkp}, w_{kjp} : Flows of product p, $p \in P$, between depots j and k, $j \in D$, $k \in D$;

Crainic, Dejax and Delorme (1989) then formulate the model as:

$$\textit{Minimize} \quad Z = \sum_{j \in D} f_j \, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} \left(c_{ijp} \, x_{ijp} + c_{jip} \, x_{jip} \right) + \sum_{j \in D} \sum_{k \in D} s_{jkp} \, w_{jkp} \right\}$$

$$\textit{Subject to} \quad \sum_{j \in D} x_{ijp} = O_{ip} \quad \text{and} \quad \sum_{j \in D} x_{jip} = D_{ip} \qquad \forall \, i \in C, p \in P \tag{1}$$

$$x_{ijp} \leq M \, y_j \quad \text{and} \quad x_{jip} \leq M \, y_j \qquad \forall \, i \in C, j \in D, p \in P \tag{2}$$

$$w_{jkp} \leq M \, y_j \quad \text{and} \quad w_{kjp} \leq M \, y_j \qquad \forall \, j \in D, k \in D, p \in P \tag{3}$$

$$\sum_{i \in C} x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} x_{jip} - \sum_{k \in D} w_{jkp} = 0 \qquad \forall \, j \in D, p \in P \tag{4}$$

$$x_{ijp} \geq 0 \quad \text{and} \quad x_{jip} \geq 0 \qquad \forall \, i \in C, j \in D, p \in P \tag{5}$$

$$w_{jkp} \geq 0 \qquad \forall \, j \in D, k \in D, p \in P \tag{6}$$

$$y_j \in \{0,1\} \qquad \forall\, j \in D. \tag{7}$$

Constraints (1) ensure that demand and supply requirements are met. Constraints (2) forbid exchanges between customers and closed depots (M is a suitably large positive constant), while constraints (3) have a similar role for interdepot movements. Constraints (4) correspond to the interdepot balancing requirements. Figure 1 illustrates the structure of the problem.

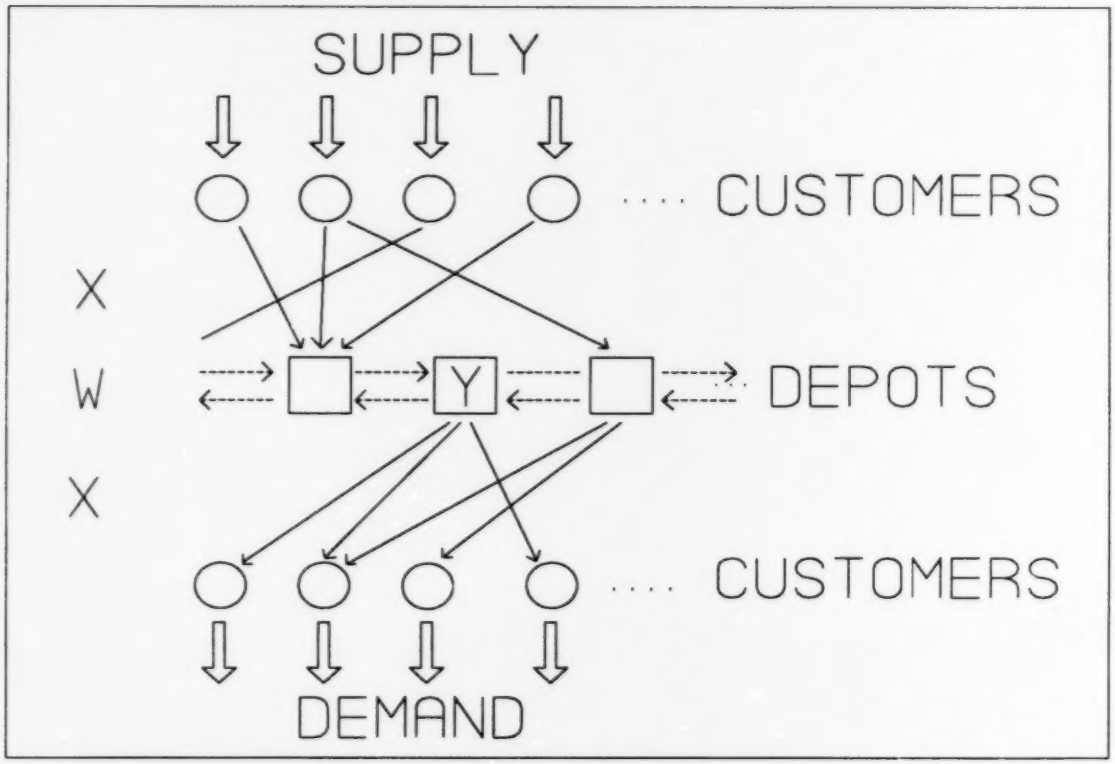

Figure 1. Structure of the SLBP

As mentioned in the introduction, we assume that for the range of distances characterizing them, interdepot movements are more efficient, per transported unit, than movements between customers and depots. This forms the rationale of the problem and, by insuring that interdepot movements do take place, contributes towards the interest and originality of the model. Indeed, Crainic, Dejax and Delorme (1989) have shown that when this hypothesis is not verified, in particular when

$$c_{ijp} + s_{jkp} \geq c_{ikp} \qquad \forall\, i \in C,\ j \in D,\ k \in D,\ p \in P,$$

there exists an optimal solution with $w_{jkp} = 0$ for all $j \in D,\ k \in D,\ p \in P$. On the other hand,

they have also shown that when interdepot transportation costs verify the triangle inequality

$$s_{jkp} \leq s_{jrp} + s_{rkp} \quad \forall\, j \in D, k \in D, r \in D$$

constraints (3) are redundant, and the model is more concisely written as:

$$(L)\ \textit{Minimize}\ \ Z = \sum_{j \in D} f_j\, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} \left(c_{ijp}\, x_{ijp} + c_{jip}\, x_{jip} \right) + \sum_{j \in D} \sum_{k \in D} s_{jkp}\, w_{jkp} \right\}$$

$$\textit{Subject to} \quad \sum_{j \in D} x_{ijp} = O_{ip} \qquad \forall\, i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = D_{ip} \qquad \forall\, i \in C, p \in P \qquad \text{(L1)}$$

$$x_{ijp} \leq O_{ip}\, y_j \qquad \forall\, i \in C, j \in D, p \in P$$

$$x_{jip} \leq D_{ip}\, y_j \qquad \forall\, i \in C, j \in D, p \in P \qquad \text{(L2)}$$

$$\sum_{i \in C} x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall\, j \in D, p \in P \qquad \text{(L3)}$$

$$x_{ijp},\, x_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P \qquad \text{(L4)}$$

$$w_{jkp} \geq 0 \qquad \forall\, j \in D, k \in D, p \in P \qquad \text{(L5)}$$

$$y_j \in \{0,1\} \qquad \forall\, j \in D \qquad \text{(L6)}$$

Since this hypothesis is rather mild and it is easily verified in practice, it is this last formulation which is usually solved.

A few algorithmic developments have already been performed for this problem. The exploration of classical heuristic approaches (greedy, interchange, etc.) has been initiated with encouraging success (Guo, 1990). Lagrangian relaxation approaches have proved quite efficient for moderate sized problems, but their performance seems to deteriorate with the number of commodities (Guo, 1990; Guo et al., 1989). Research is continuing in this direction, as well as in the area of primal/dual approaches (Tétreault, 1990). Finally, Crainic, Delorme and Dejax (1991) propose a branch-and-bound algorithm for the problem. They show, in particular, that the classical results for the simple plant location problem (Krarup and Pruzan, 1983) do not extend to the SLBP. Indeed, the branching rules which are efficient in the classical case are among the worst for the present case. The authors then proceed to develop and analyze specific rules, and obtain an efficient branch-and-bound framework. We use their results to analyze the precision of the dual-ascent procedure.

II. DUAL-ASCENT ALGORITHMS

Let us consider the *strong linear relaxation* of the *normalized* version of problem ($\mathcal{L}$):

(*P*) *Minimize* $Z(y, x, w) =$

$$\sum_{j \in D} f_j \, y_j + \sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (C_{ijp} \, x_{ijp} + C_{jip} \, x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp} \, w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall \, i \in C, p \in P \qquad (u_{ip})$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall \, i \in C, p \in P \qquad (v_{ip}) \qquad \text{(P1)}$$

$$x_{ijp} \le y_j \qquad \forall \, i \in C, j \in D, p \in P \qquad (\gamma_{ijp})$$

$$x_{jip} \le y_j \qquad \forall \, i \in C, j \in D, p \in P \qquad (\gamma_{jip}) \qquad \text{(P2)}$$

$$\sum_{i \in C} O_{ip} \, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip} \, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \qquad (\lambda_{jp}) \qquad \text{(P3)}$$

$$\forall \, j \in D, p \in P$$

$$x_{ijp}, x_{jip} \ge 0 \qquad \forall \, i \in C, j \in D, p \in P \qquad \text{(P4)}$$

$$w_{jkp} \ge 0 \qquad \forall \, j, k \in D, p \in P \qquad \text{(P5)}$$

$$y_j \ge 0 \qquad \forall \, j \in D \qquad \text{(P6)}$$

where $C_{ijp} = O_{ip} c_{ijp}$ and $C_{jip} = D_{ip} c_{jip}$, $\forall \; i \in C, j \in D, p \in P$. Let (y^*, x^*, w^*) be an optimal solution of problem (P), and $Z(y^*, x^*, w^*)$ the corresponding value of the objective function.

The dual formulation of the above problem may be stated as follows:

(*D*) *Maximize* $Z(u, v, \gamma, \lambda) = \sum_{p \in P} \sum_{i \in C} (u_{ip} + v_{ip})$

Subject to

$$u_{ip} - \gamma_{ijp} + O_{ip} \, \lambda_{jp} \le C_{ijp} \qquad \forall \, i \in C, j \in D, p \in P$$

$$v_{ip} - \gamma_{jip} - D_{ip} \, \lambda_{jp} \le C_{jip} \qquad \forall \, i \in C, j \in D, p \in P \qquad \text{(D1)}$$

$$\sum_{p \in P} \sum_{i \in C} (\gamma_{ijp} + \gamma_{jip}) \le f_j \qquad \forall \, j \in D \qquad \text{(D2)}$$

$$\lambda_{jp} - \lambda_{kp} \le s_{kjp} \qquad \forall\, j,k \in D,\ p \in P \tag{D3}$$

$$\gamma_{ijp},\ \gamma_{jip} \ge 0 \qquad \forall\, i \in C,\ j \in D,\ p \in P \tag{D4}$$

In this formulation, u_{ip} and v_{ip} are the dual variables associated to the supply and demand constraints, respectively, for each customer and for each commodity. Dual variables γ_{ijp} and γ_{jip} correspond to the "location feasibility" constraints (P2) for each direction of operation between a depot and a customer, while the λ_{jp} variables are associated to the balancing constraints for each commodity at each depot.

2.1 Dual Properties and Subproblems

To solve our problem, we intend to follow a strategy which consists in iteratively modifying the values of the dual variables in order to increase the value of the dual objective $Z(u,v,\gamma,\lambda)$. This allows us to efficiently compute lower (and upper) bounds on the value of the objective function of the original problem, bounds which may then be used either to guide an enumeration algorithm, or to qualify heuristic-obtained solutions.

Observe that, when the λ_{jp} variables are set to values $\bar{\lambda}_{jp}$ which verify

$$\bar{\lambda}_{jp} - \bar{\lambda}_{kp} \le s_{kjp} \qquad \forall\, j,k \in D,\ p \in P,$$

the following formulation is obtained:

$$(D_1(\bar{\lambda}))\quad \textit{Maximize}\quad Z(u,v,\gamma \mid \bar{\lambda}) = \sum_{p \in P} \sum_{i \in C} (u_{ip} + v_{ip})$$

Subject to

$$u_{ip} - \gamma_{ijp} \le (C_{ijp} - O_{ip}\,\bar{\lambda}_{jp}) \qquad \forall\, i \in C,\ j \in D,\ p \in P$$

$$v_{ip} - \gamma_{jip} \le (C_{jip} + D_{ip}\,\bar{\lambda}_{jp}) \qquad \forall\, i \in C,\ j \in D,\ p \in P$$

$$\sum_{p \in P} \sum_{i \in C} (\gamma_{ijp} + \gamma_{jip}) \le f_j \qquad \forall\, j \in D$$

$$\gamma_{ijp},\ \gamma_{jip} \ge 0 \qquad \forall\, i \in C,\ j \in D,\ p \in P$$

The dual of this formulation is

$(P_1(\bar{\lambda}))$ *Minimize* $Z(y, x \mid \bar{\lambda}) =$

$$\sum_{j \in D} f_j y_j + \sum_{p \in P} \sum_{i \in C} \sum_{j \in D} \left\{ (C_{ijp} - O_{ip} \bar{\lambda}_{jp}) x_{ijp} + (C_{jip} + D_{ip} \bar{\lambda}_{jp}) x_{jip}) \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \quad \text{and} \quad \sum_{j \in D} x_{jip} = 1 \qquad \forall i \in C, p \in P$$

$$x_{ijp} \leq y_j \quad \text{and} \quad x_{jip} \leq y_j \qquad \forall i \in C, j \in D, p \in P$$

$$x_{ijp} \geq 0 \quad \text{and} \quad x_{jip} \geq 0 \qquad \forall i \in C, j \in D, p \in P$$

$$y_j \geq 0 \qquad \forall j \in D$$

This formulation clearly is the strong linear relaxation of a simple plant location problem (Krarup and Pruzan, 1983). Here, the multicommodity aspect of the problem, as well as the fact that customers may both send to and receive items from depots, does not increase the difficulty of the problem. It does, however, affect the implementation of the solution procedures and it increases the data storage requirements.

The $\bar{\lambda}$ variables characterize the state of "unbalance" of the system at each depot. In particular, when $\bar{\lambda}_{jp}$ is relatively large and positive, depot j attracts more flow then it sends to customers, since the transportation costs associated with x_{ijp} in problem $P_1(\bar{\lambda})$ increase while those of variables x_{jip} decrease. The same is true, mutatis mutandis, for large, negative values of $\bar{\lambda}_{jp}$. Thus, large values for some $\bar{\lambda}$ tend to indicate that the corresponding depots have important shortages or surpluses of certain commodities, which result in interdepot balancing flows.

Problems $D_1(\bar{\lambda})$ and $P_1(\bar{\lambda})$ and their solutions may be used to determine a feasible solution to problem (P) and to compute lower and upper bounds on $Z(y^*, x^*, w^*)$. Let $(\bar{y}, \bar{x})$ be the optimal solution of problem $P_1(\bar{\lambda})$, and $Z(\bar{y}, \bar{x} \mid \bar{\lambda})$ the corresponding objective value. Consider

$(P(\bar{y}, \bar{x}, w))$ *Minimize* $Z(w \mid \bar{y}, \bar{x}) = \sum_{p \in P} \sum_{j \in D} \sum_{k \in D} s_{jkp} w_{jkp}$

$$\text{Subject to} \quad \sum_{k \in D} w_{kjp} - \sum_{k \in D} w_{jkp} = \sum_{i \in C} D_{ip} \bar{x}_{jip} - \sum_{i \in C} O_{ip} \bar{x}_{ijp} \qquad \forall j \in D, p \in P$$

$$w_{jkp} \geq 0 \quad \forall j, k \in D, p \in P$$

which is the original problem (P) with variables y and x set to values $\bar{y}$ and $\bar{x}$, respectively. Problem $(P(\bar{y}, \bar{x}, w))$ evidently is a multicommodity transportation problem, which decomposes into l mutually exclusive minimum cost transportation problems, one for each commodity. The origin and destination nodes of this problem correspond to the open depots in the optimal solution to problem $(P_1(\bar{\lambda}))$, while the corresponding supplies and demands are computed according to the allocation of customers to depots defined by this same optimal solution. Let $\hat{w}$ be its optimal solution and $Z(\bar{y}, \bar{x}, \hat{w})$ the corresponding objective function value. Then

Proposition 1.

$$Z(\bar{y}, \bar{x} \mid \bar{\lambda}) \leq Z(y^*, x^*, w^*) \leq Z(\bar{y}, \bar{x}, \hat{w}) .$$

To justify this result, note that $(\bar{y}, \bar{x}, \hat{w})$ form a feasible solution to problem (P) and, thus, $Z(y^*, x^*, w^*) \leq Z(\bar{y}, \bar{x}, \hat{w})$. The other side of the proposition follows directly from linear duality arguments.

A possibly tighter bound may however be derived. Note that a feasible solution to problem (P) may also be obtained by setting the y variables to $\bar{y}$ and then by solving

$(P(\bar{y}, x, w))$ *Minimize* $Z(w, x \mid \bar{y}) =$

$$\sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} (C_{ijp} x_{ijp} + C_{jip} x_{jip}) + \sum_{j \in D} \sum_{k \in D} s_{jkp} w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall i \in C, p \in P$$

$$\sum_{i \in C} O_{ip} x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip} x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall j \in D, p \in P$$

$$\bar{y}_j \geq x_{ijp}, x_{jip} \geq 0 \qquad \forall i \in C, j \in D, p \in P$$

$$w_{jkp} \geq 0 \qquad \forall\, j,k \in D, p \in P$$

Problem $(P(\bar{y}, x, w))$ is a multicommodity minimum cost network flow problem, which also decomposes into l mutually exclusive minimum cost transshipment problems. Furthermore, it is a relaxation of problem $(P(\bar{y}, \bar{x}, w))$, in the sense that it considers the same open depots but it allows the reassignment of customers to depots. Let $(\bar{x}, \bar{w})$ be the optimal solution of $(P(\bar{y}, x, w))$ and $Z(\bar{y}, \bar{\bar{x}}, \bar{\bar{w}})$ the corresponding objective function value. We then have

Proposition 2.

$$Z(\bar{y}, \bar{x} \mid \bar{\lambda}) \leq Z(y^*, x^*, w^*) \leq Z(\bar{y}, \bar{\bar{x}}, \bar{\bar{w}}) \leq Z(\bar{y}, \bar{x}, \hat{w}) .$$

Note that similar bounds, although less tight, may be derived for any feasible solution (y, x) to problem $(P_1(\bar{\lambda}))$. We use this fact in the solution procedures presented at the end of the section.

A second set of bounds on $Z(y^*, x^*, w^*)$ may be derived from (D). Observe that, when variables γ_{ijp} and γ_{jip} are fixed to values $\tilde{\gamma}_{ijp}$ and $\tilde{\gamma}_{jip}$ which verify

$$\sum_{p \in P} \sum_{i \in C} (\tilde{\gamma}_{ijp} + \tilde{\gamma}_{jip}) \leq f_j \qquad \forall\, j \in D$$

$$\tilde{\gamma}_{ijp}, \tilde{\gamma}_{jip} \geq 0 \qquad \forall\, i \in C, j \in D, p \in P ,$$

the original dual problem (D) reduces to

$(D_2(\tilde{\gamma}))$ *Maximize* $Z(u, v, \lambda \mid \tilde{\gamma}) = \sum_{p \in P} \sum_{i \in C} (u_{ip} + v_{ip})$

Subject to

$$u_{ip} + O_{ip}\, \lambda_{jp} \leq C_{ijp} + \tilde{\gamma}_{ijp} \qquad \forall\, i \in C, j \in D, p \in P$$

$$v_{ip} - D_{ip}\, \lambda_{jp} \leq C_{jip} + \tilde{\gamma}_{jip} \qquad \forall\, i \in C, j \in D, p \in P$$

$$\lambda_{jp} - \lambda_{kp} \leq s_{kjp} \qquad \forall\, j,k \in D, p \in P$$

The dual of the above problem is

$(P_2(\tilde{\gamma}))$ *Minimize* $Z(x, w \mid \tilde{\gamma}) =$

$$\sum_{p \in P} \left\{ \sum_{i \in C} \sum_{j \in D} \left(\left(C_{ijp} + \tilde{\gamma}_{ijp} \right) x_{ijp} + \left(C_{jip} + \tilde{\gamma}_{jip} \right) x_{jip} \right) + \sum_{j \in D} \sum_{k \in D} s_{jkp} \, w_{jkp} \right\}$$

Subject to

$$\sum_{j \in D} x_{ijp} = 1 \qquad \forall \, i \in C, p \in P$$

$$\sum_{j \in D} x_{jip} = 1 \qquad \forall \, i \in C, p \in P$$

$$\sum_{i \in C} O_{ip} \, x_{ijp} + \sum_{k \in D} w_{kjp} - \sum_{i \in C} D_{ip} \, x_{jip} - \sum_{k \in D} w_{jkp} = 0 \quad \forall \, j \in D, p \in P$$

$$x_{ijp}, x_{jip} \geq 0 \qquad \forall \, i \in C, j \in D, p \in P$$

$$w_{jkp} \geq 0 \qquad \forall \, j, k \in D, p \in P$$

which is a multicommodity uncapacitated minimum cost network flow formulation (Crainic, Dejax and Delorme, 1989).

Note the complementarity slackness conditions between constraints (P2) and the γ variables: for all $i \in C$, $j \in D$ and $p \in P$

$$\gamma_{ijp} > 0 \implies y_j = x_{ijp} \quad \text{and} \quad \gamma_{jip} > 0 \implies y_j = x_{jip}$$

and

$$y_j > x_{ijp} \implies \gamma_{ijp} = 0 \quad \text{and} \quad y_j > x_{jip} \implies \gamma_{jip} = 0.$$

Thus, in problem $P_2(\tilde{\gamma})$, the $\tilde{\gamma}_{ijp}$ and $\tilde{\gamma}_{jip}$ terms adjust the costs of transportation between customers and depots to account for the fixed, opening and operating, cost of each open depot j. Accordingly, one may interpret these terms as prices to be paid in order to increase the value of y_j, hence to open depot j.

Let $(\tilde{x}, \tilde{w})$ be the solution of problem $(P_2(\tilde{\gamma}))$ and $Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma})$ the corresponding objective function value. Define $\tilde{y}$ as follows:

$$\tilde{y}_j = \underset{i \in C, p \in P}{\textit{maximum}} \{ \tilde{x}_{ijp}, \tilde{x}_{jip} \} \quad \forall \, j \in D.$$

Then, $\tilde{y}_j \leq 1$, $\forall\, j \in D$, and $(\tilde{y}, \tilde{x}, \tilde{w})$ is a feasible solution for problem (P). Consequently, by invoking the usual linear duality arguments, we have

Proposition 3.

$$Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma}) \leq Z(y^*, x^*, w^*) \leq Z(\tilde{y}, \tilde{x}, \tilde{w}).$$

2.2 Dual-Ascent Procedures

The results of the previous subsection may be used to build several dual-ascent procedures to solve problem (P). In particular, problems $(P_1(\bar{\lambda}))$ and $(P_2(\tilde{\gamma}))$ may be solved alternatively, the results of each problem being assigned as input to the other. A complete iteration of such a procedure consists of the sequential resolution of both problems, the particular procedure being defined by the specific order of resolution of the two problems. Define *LB* as the current lower bound, *BUB* the best current upper bound, and

Procedure *FLIP*$(\tilde{\gamma})$ – Minimum Cost Multicommodity Flow Problem

- Solve problem $(P_2(\tilde{\gamma}))$; let $(\tilde{x}, \tilde{w})$ be the optimal solution, and $(\tilde{u}, \tilde{v}, \tilde{\lambda})$ the corresponding optimal dual variables;
- Compute $\tilde{y}_j = \underset{i \in C,\, p \in P}{maximum} \{ \tilde{x}_{ijp}, \tilde{x}_{jip} \}$ $\forall\, j \in D$;
- $LB = Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma})$; $BUB = minimum \{ BUB, Z(\tilde{y}, \tilde{x}, \tilde{w}) \}$.

and

Procedure *FLOP*$(\bar{\lambda})$ – Uncapacitated Multicommodity Location Problem

- Solve problem $(P_1(\bar{\lambda}))$; let $(\bar{y}, \bar{x})$ be the optimal solution, and $(\bar{u}, \bar{v}, \bar{\gamma})$ the corresponding optimal dual variables;
- Solve problem $P(\bar{y}, x, w)$;
- $LB = Z(\bar{y}, \bar{x} \mid \bar{\lambda})$; $BUB = minimum \{ BUB, Z(\bar{y}, \bar{\bar{x}}, \bar{\bar{w}}) \}$.

We may now formally state the two dual-ascent procedures for problem (P), where ε_1 and ε_2 are suitably small real numbers, and *maxiter* is an integer representing the maximum

number of iterations allowed:

Procedure *FLIP-FLOP*

[1] $\tilde{\gamma} = 0$;
$BLB = 0$;
$BUB = \infty$.

[2] $LBLB = BLB$.

[3] Solve the *FLIP* problem.

[4] $\bar{\lambda} = \tilde{\lambda}$.

[5] Solve the *FLOP* problem.

[6] $\tilde{\gamma} = \bar{\gamma}$.

[7] $BLB = LB$

[8] If $\frac{BLB - LBLB}{LBLB} < \varepsilon_1$ or,
$GAP = \frac{BUB - BLB}{BLB} < \varepsilon_2$ or,
the number of performed iterations
is equal to *maxiter*,
then STOP; otherwise go to step 2.

Procedure *FLOP-FLIP*

[1] $\bar{\lambda} = 0$;
$BLB = 0$;
$BUB = \infty$.

[2] $LBLB = BLB$.

[3] Solve the *FLOP* problem.

[4] $\tilde{\gamma} = \bar{\gamma}$.

[5] Solve the *FLIP* problem.

[6] $\bar{\lambda} = \tilde{\lambda}$.

[7] $BLB = LB$

[8] If $\frac{BLB - LBLB}{LBLB} < \varepsilon_1$ or,
$GAP = \frac{BUB - BLB}{BLB} < \varepsilon_2$ or,
the number of performed iterations
is equal to *maxiter*,
then STOP; otherwise go to step 2.

Note that, when in the *FLOP* procedure problem $(P_1(\bar{\lambda}))$ is solved to optimality, $(\bar{y}, \bar{x})$ and $(\bar{u}, \bar{v}, \bar{\gamma})$ form a pair of primal-dual variables, and the two procedures compute an increasing series of lower bounds. In practice, however, it is not reasonable to solve to optimality a multicommodity location problem at each iteration. Indeed, in our implementation, we only obtain (see the next section) "good" feasible solutions to problems $(D_1(\bar{\lambda}))$ and $(P_1(\bar{\lambda}))$ (to avoid overcharging the paper, we keep the same notation for these solutions). Then, to ensure the monotonicity of the lower bound values, the $\bar{u}$ and $\bar{v}$ variables obtained at the end of the *FLOP* procedure are used as initial values for the minimum cost multicommodity network flow problem *FLIP*.

Finally, note that the bounds might be further tightened if one remarks that, following the resolution of $(P_1(\bar{\lambda}))$, the values of the γ variables may be somewhat changed without modifying the optimal solution of the dual problem $(D_1(\bar{\lambda}))$. Indeed, these variables do not appear in the objective function of this problem. Thus, a modification which does not affect its realisability has no impact on its optimality either. But, with higher values for the γ variables, the value of $Z(\tilde{x}, \tilde{w} \mid \tilde{\gamma})$, corresponding to the optimal objective value of problem $(P_2(\tilde{\gamma}))$, increases; hence, a tighter lower bound computed at the end of the *FLIP* procedure.

The modification of the γ variables thus aims to make the transportation costs between customers and depots (on the existing links) to reflect more closely the cost of opening and operating the depots. Consequently, the values of these variables are increased to take out any slack which may exist at the end of the procedure *FLOP* in the dual constraints (D2). Let e_j, $\forall\, j \in D$, be the slack variables in constraints (D2). For the optimum solution of problem $(P_1(\bar{\lambda}))$, we then have

$$\sum_{p \in P} \sum_{i \in C} (\bar{\gamma}_{ijp} + \bar{\gamma}_{jip}) + \bar{e}_j = f_j \qquad \forall\, j \in D.$$

Step [6] ([4]) in procedure *FLIP-FLOP (FLOP-FLIP)* then becomes

$$\tilde{\gamma}_{ijp} = \bar{\gamma}_{ijp} + \bar{e}_j \frac{O_{ip}}{\sum_{i \in C,\, p \in P \mid C_{ip} < \infty} O_{ip} + \sum_{i \in C,\, p \in P \mid C_{jp} < \infty} D_{ip}}$$

$$\tilde{\gamma}_{jip} = \bar{\gamma}_{jip} + \bar{e}_j \frac{D_{ip}}{\sum_{i \in C,\, p \in P \mid C_{ip} < \infty} O_{ip} + \sum_{i \in C,\, p \in P \mid C_{jp} < \infty} D_{ip}}$$

III. EXPERIMENTATION

The experimentation of the dual-ascent based procedures was conducted on two sets of "random" test problems, and on data based on an actual large scale application. We generate the random test problems by using the procedure described by Crainic, Delorme and Dejax (1991), who also used some of the same problems to test enumeration algorithms for the location-allocation problem with balancing requirements.

The aim of the experiments is to determine the behaviour, precision and relative performance of the two dual-ascent heuristics for various types of problems. The main problem characteristics considered are the number of candidate depots and the network dimensions, the number of products, the magnitude of the interdepot traffic generated by the differences between the supply and demand present in each one of the zones of the network, and the relative importance of the fixed depot costs versus the transportation costs. The principal descriptors used to quantify the performance of the two procedures are the speed of convergence, the computation time, and the quality of the solution measured both as the relative gap between the lower and upper bounds, and as the difference between the best upper bound and the optimal solution.

Note that our main goal was not to obtain the fastest procedure, but rather to determine the most appropriate one among the available alternatives, given the problem characteristics. Consequently, we did not program our own minimum cost network flow algorithm for the FLIP subproblem; instead, we adapted the RNET code (Grigoriadis and Hsu, 1979) to take into account real valued flows and costs (computed in double precision). We have coded, however, a multicommodity version of DUALOC (Erlenkotter, 1978), in order to solve the FLOP subproblem. The "dual ascent" phase of the algorithm follows Erlenkotter, while the "adjustment" step is based on the work of Van Roy and Erlenkotter (1982). Note that we compute only a "good" solution solution, rather than the optimal one, to the location subproblem. Hence, the remarks on the previous section apply. The procedures are coded in FORTRAN and the experiments were run on a network of SUN Sparc workstations.

Table 1 displays the typical dimensions of the random-generated test problems that we use. The type of the problem is described by its label: "m" and "l" indicate medium and large sized problems respectively, N represents the degree of unbalance present in the system

Type of Problem	Regular Nodes	Candidate Locations	Regular Arcs (1 Product)	Interdepot Arcs (1 Product)
mz*N* p*P*	125	25	1760	600
lz*N* p*P*	219	44	5260	1892

Table 1. Typical Dimensions for Random Generated Test Problems

(the higher the figure, the larger the required interdepot flows), while *P* corresponds to the number of products. Note that, for a specific problem, the figures indicated in Table 1 may vary by a few units. Also, for multicommodity problems, the figures for regular and interdepot arcs are multiplied by the number of products involved.

To complete our experiments, we also used data from an actual application. The original application considers the planning of the land transportation operations of a major, Europe based, international container shipping company. The problem consists of 12 aggregated types of products, 299 aggregated customer zones, XX candidate depots, in a geographical zone covering five western European countries. Once the 9 initial transportation modes have been aggregated into the least cost (eventually multimodal) arcs between each pair of nodes, we obtained a network that contains 4197 interdepot arcs and 32733 arcs between customers and depots. A detailed analysis of this case may be found in Dejax, Crainic and Delorme (1991). In the present paper, we concentrate on the computational performance of the dual-ascent procedures when used to solve this problem. Hence, since the original fixed depot cost are rather low, we also solved problems were the fixed costs were multiplied by 10, 100 and 1000, respectively.

Table 2 displays the results obtained by using the two procedures to solve 42 medium-sized problems. The label 10x indicates that we solve again the problem defined on the previous line but with the fixed depot costs multiplied tenfold. For each problem and procedure, the table indicates the iteration where the leveling off of the lower bound occurs, the corresponding computation time (in CPU seconds), the relative gap (in percentage), and the difference (in percentage) between the objective value of the best feasible solution found (the best upper bound) and the optimal solution of the problem. The results of the tests performed on large sized problems and on those based on real life data are shown in Table 3.

Problem	FLIP–FLOP				FLOP–FLIP			
	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.
mz1p1	1	0.65	0.66	0.00	2	1.48	1.74	0.10
10x	2	3.56	6.93	6.10	2	2.83	5.44	3.87
mz1p2	2	2.99	1.64	0.38	2	1.52	5.64	2.53
10x	2	13.29	6.10	1.90	3	13.58	7.14	1.90
mz1p3	1	1.26	0.67	0.16	3	5.63	1.88	0.32
10x	2	19.40	1.30	0.00	3	22.97	5.20	0.90
mz1p4	1	2.53	1.30	0.00	2	5.63	0.74	0.00
10x	2	12.66	3.01	1.36	3	14.53	7.62	1.71
mz1p5	1	2.73	0.66	0.18	3	12.55	1.47	0.14
10x	2	20.75	1.62	0.00	3	8.54	4.54	0.30
mz1p6	1	4.31	0.30	0.04	2	10.30	0.70	0.01
10x	2	22.80	0.45	0.00	2	24.38	2.15	0.00
mz1p10	1	6.91	0.43	0.05	2	16.23	0.70	0.04
10x	2	100.93	0.98	0.09	3	63.96	3.36	0.34
mz2p1	2	1.54	2.30	1.07	3	2.63	5.78	0.54
10x	4	6.10	11.73	2.82	2	1.94	18.29	2.82
mz2p2	2	3.75	1.51	0.00	3	3.28	3.90	0.48
10x	2	9.80	9.05	3.98	3	9.32	11.58	0.00
mz2p3	2	5.81	1.13	0.50	3	6.95	1.97	0.16
10x	2	13.82	0.78	0.00	3	16.16	6.78	1.02
mz2p4	1	2.24	0.15	0.00	2	6.17	0.66	0.17
10x	2	14.89	2.10	0.94	3	18.40	4.28	0.81
mz2p5	1	2.72	0.97	0.13	3	12.76	1.82	0.16
10x	2	7.61	3.71	0.60	3	7.02	7.86	0.88
mz2p6	1	2.84	0.27	0.06	2	8.34	0.51	0.05
10x	2	39.22	2.07	1.39	3	50.51	2.00	0.19
mz2p10	1	5.15	0.15	0.00	2	18.64	0.61	0.11
10x	2	83.29	1.25	0.69	3	76.68	3.02	0.89
mz4p1	2	1.25	1.85	1.29	3	1.54	6.04	1.54
10x	3	5.35	4.36	1.08	2	2.78	6.40	2.44
mz4p2	2	3.69	0.87	0.03	3	3.23	3.23	0.03
10x	3	7.33	2.01	0.00	3	3.48	6.59	0.00
mz4p3	1	2.20	0.22	0.01	3	7.92	0.70	0.33
10x	2	11.36	2.60	0.91	3	19.23	8.40	2.38
mz4p4	2	6.25	0.65	0.37	3	9.05	1.43	0.54
10x	2	20.05	2.41	0.75	3	10.59	6.79	0.69
mz4p5	1	2.02	0.37	0.23	3	12.39	0.79	0.12
10x	2	20.22	2.25	0.57	3	7.67	6.09	0.51
mz4p6	1	3.22	0.75	0.35	3	10.17	1.14	0.42
10x	2	26.60	1.66	0.00	3	18.02	5.47	0.61
mz4p10	1	5.95	0.19	0.00	2	14.56	0.40	0.00
10x	2	97.43	1.08	0.00	3	70.95	4.64	1.21

Table 2. Results for Random Problems of Medium Size

The first main conclusion of the experimentation is that the two procedures converge extremely fast. We evaluate the speed of convergence of the dual-based procedures by measuring how fast the relative improvement in the value of the lower bound (test performed in step 8 of the procedures) levels off. Indeed, in almost all cases, less than 1% improvement in the lower bound is obtained after the first or the second iteration, for the FLIP-FLOP procedure. The FLOP-FLIP procedure generally requires two or three iterations. These results were observed for all our tests, irrespective of the problem size or complication. On the other hand, in practical terms, no further improvement may be expected by performing more iterations. Indeed, we run the algorithms for ten iterations on all problems and observed either negligible improvements, or no improvement at all.

Problem	γ not adjusted				γ adjusted: $e_j = 0$				γ adjusted: $e_j = f_j$			
	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.	Iter.	Time	Gap	Diff.
lz1p1	1	4.83	0.57	0.27	1	4.86	0.57	0.27	1	5.53	0.77	0.33
10x	2	44.00	29.45	25.27	2	49.23	24.38	20.45	3	37.47	3.38	0.16
lz1p2	2	22.37	1.12	0.12	2	18.38	0.98	0.12	2	15.67	1.11	0.06
10x	1	22.37	0.90	0.00	1	22.51	0.90	0.00	1	28.22	0.86	0.00
lz3p1	2	6.44	1.34	1.00	2	7.17	1.34	1.00	2	6.76	1.46	1.04
10x	2	21.82	2.70	1.27	2	22.23	2.31	1.27	2	17.76	2.10	0.00
lz3p2	2	20.26	0.96	0.15	2	18.65	0.83	0.13	2	16.69	0.97	0.19
10x	2	62.81	3.15	0.56	2	55.21	2.94	0.38	2	54.32	3.52	0.38
lz5p1	2	6.70	0.77	0.12	2	6.76	1.01	0.44	2	6.71	1.05	0.38
10x	2	12.06	3.62	2.22	2	10.85	3.54	2.22	3	22.82	1.97	0.00
lz5p2	2	16.72	0.68	0.23	2	16.09	0.64	0.19	2	21.72	1.58	1.07
10x	2	22.02	4.94	0.00	3	32.30	3.14	0.00	2	37.10	3.33	0.00
original	1	4.76	0.00	0.00	1	4.82	0.00	0.00	1	46.47	0.10	0.03
10x	1	8.54	0.21	0.03	1	8.78	0.21	0.03	1	38.79	0.48	0.03
100x	2	27.77	1.47	0.16	2	47.71	0.87	0.00	2	60.43	1.17	0.16
1000x	2	47.68	0.95	0.00	2	58.61	0.43	0.00	2	80.81	1.03	0.01

Table 3. FLIP-FLOP with Gamma Adjustments

The two procedures require relatively limited computational efforts. We observe that, as expected, computation times increase with the number of products; the effect of the system unbalance factor, as well as that of the problem size, is less marked. One remarks, however, that more "complicated" problems (i.e., more zones and more products) are comparatively easier to solve than simpler ones. This is similar to, but rather less extreme than, the behaviour of branch-and-bound procedures for the same type of problems (Crainic, Delorme

and Dejax; 1991). One also notes that the amplitude of the fixed costs has a significant impact on the computation time. This is to be expected, since the (continuous) minimum cost network flow problem cannot account very precisely for the fixed depot costs.

The two procedures achieve excellent solutions, as illustrated by the relative gaps achieved which are, in general, relatively small. Furthermore, when compared with the results achieved, for the same problems, in 1000 iterations of a branch-and-bound procedure based on the weak relaxation of the problem (Crainic, Delorme and Dejax; 1991), the current tests suggest that, by using dual-ascent heuristics, the gap may be improved very significantly (between 20% and 95% for medium sized problems). This is verified even for "difficult" problems, such as mz2p1x10 for which the FLOP-FLIP procedure gives a near optimal solution and a gap some 55% lower than that of the branch-and-bound solution.

One also notes that, in general, the FLIP-FLOP procedure gives more precise solutions than the FLOP-FLIP algorithm. On the other hand, the magnitude of the fixed costs have a definitive impact on the quality of the solution obtained: the relative gap tends to increase for higher fixed costs.

Worth noting is the fact that the procedures achieve solutions which are optimal or very near the optimal. Optimal solutions were obtained by using a branch-and-bound procedure that uses the dual-ascent heuristics to evaluate the lower and upper bounds (Gendron and Crainic, 1991). As the figures displayed in Tables 2 and 3 illustrate, the (feasible) solution that corresponds to the best upper bound is often optimal, or has a very low optimality gap. Note, however, that the solution which corresponds to the best lower bound is never feasible (integer). Finally, more optimal solutions are obtained by using the FLIP-FLOP procedure. It is the same procedure which generally achieves the smaller optimality gaps. Hence, in general, the FLIP-FLOP procedure is to be preferred to the FLOP-FLIP one: computation times are lower, the solution is obtained more rapidly (in terms of number of iterations), while its quality is superior (in terms of the relative and optimality gaps).

These observations were further corroborated by the results of the second set of experiments. This second experimental phase is aimed at evaluating the impact on the general behaviour and performance of the dual-ascent heuristics of the adjustments to the γ variables (in steps [6] and [4] of procedures FLIP-FLOP and FLOP-FLIP, respectively) to tighten the

bounds. Table 3 displays these results for the large size problems, as well as for those derived from the actual application (identified as "original"), for the FLIP-FLOP procedure. Note that two initial adjustments are possible for the "slack" (e_j) variables: at zero or at the fixed cost. The respective results are given in the second and third groups of columns of Table 3.

We note that the calculation and utilization of the adjusted γ variables increases slightly the computational effort needed to solve the problems. The reward, however, is a sharp reduction of the relative gap, and an increase in the number of problems solved to optimality. This is especially true when the slack variables are initially adjusted at the fixed costs values. Hence, our conclusion is that the FLIP-FLOP procedure with the adjustment of the γ variables, with initial slacks at fixed costs, should be used in all cases.

It is interesting to note that the FLIP-FLOP dual-adjustment based procedure has solved to optimality, and in only one iteration, the problem derived from the real application. This emphasizes the behaviour of this type of solution method when applied to the class of location-distribution problems with balancing requirements. The method behaves better when one of its two components, the location and the minimum cost subproblems, "dominates" the other. For example, in the original problem, fixed costs are very low, hence the flow problem is the most important. On the other hand, multiplying the fixed costs by 1000, makes location the determinant component of the problem, and the optimal solution is obtained. This behaviour is also verified by a simple experiment. We multiplied the costs of the interdepot arcs by 100. Then, the hypothesis mentioned in Section I does no longer hold, and interdepot balancing movements are no longer interesting. Hence, the problem is now almost equivalent to a location problem and the optimal solution is generally found in one iteration.

Finally, let's mention that the FLOP-FLIP procedure displays the same general behaviour. Its poorer performance may be explained by the fact that it starts by solving a location problem. Thus, the balancing factor is not taken into account from the beginning. In particular, some depots may be closed even if they might be required to globally balance the system. Hence, the procedure tends towards a less good solution in almost all circumstances.

CONCLUSION

We have presented a dual-ascent based approach for solving multicommodity location-allocation problems with balancing requirements. Two variants have been described and analyzed and both have been shown to solve very efficiently a large gamut of problem types, including problems from actual applications. The FLIP-FLOP procedure, however, consistently outperforms the other, and should therefore be prefered. Used as an heuristic, the procedure yields very good solutions, and often achieves optimality.

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